

Mathematical studies
Standard level
Paper 2

Thursday 3 May 2018 (morning)

1 hour 30 minutes

Instructions to candidates

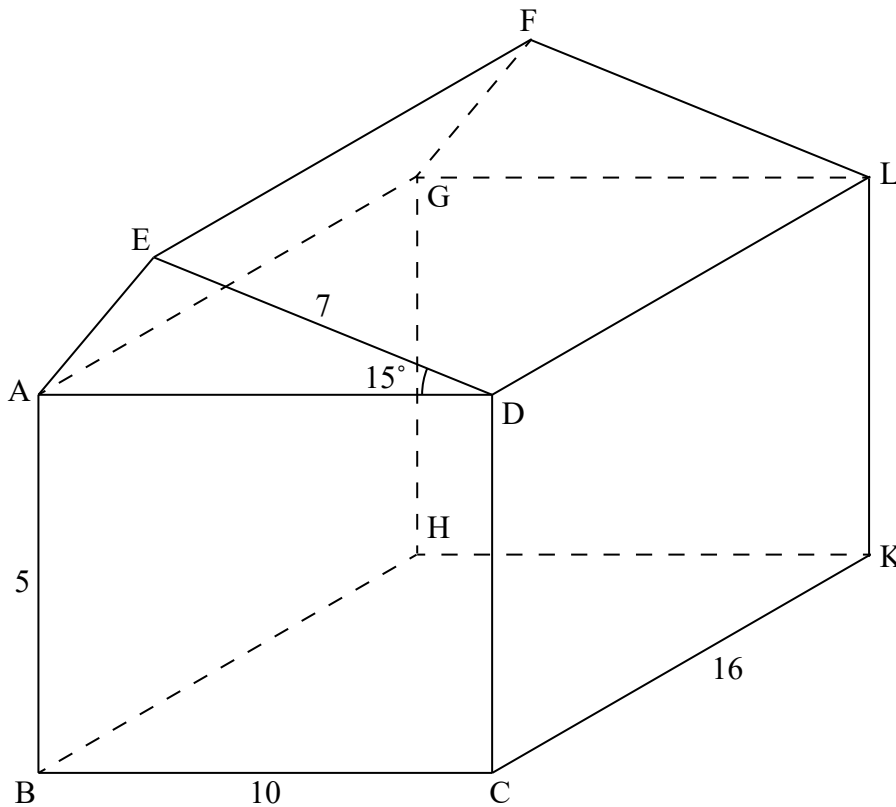
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all the questions in the answer booklet provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

Answer **all** questions in the answer booklet provided. Please start each question on a new page. You are advised to show all working, where possible. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. [Maximum mark: 18]

Farmer Brown has built a new barn, on horizontal ground, on his farm. The barn has a cuboid base and a triangular prism roof, as shown in the diagram.

diagram not to scale



The cuboid has a width of 10 m, a length of 16 m and a height of 5 m.
 The roof has two sloping faces and two vertical and identical sides, ADE and GLF.
 The face DEFL slopes at an angle of 15° to the horizontal and $ED = 7$ m.

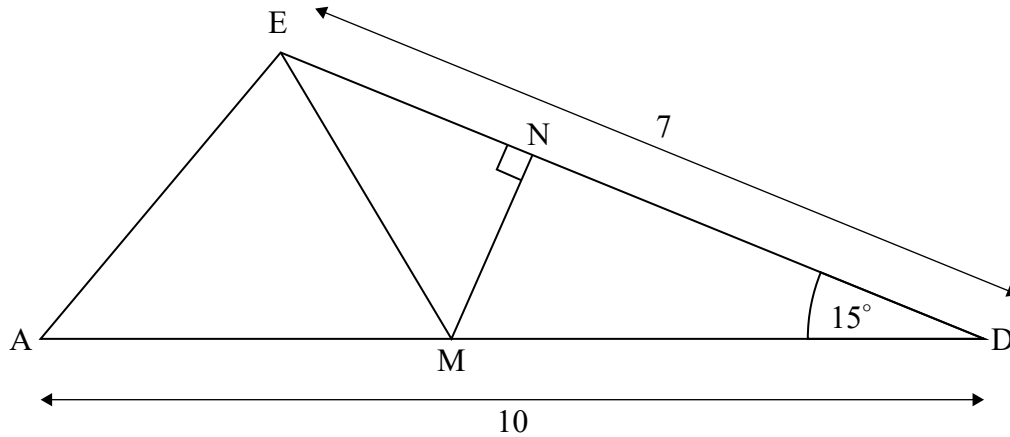
- (a) Calculate the area of triangle EAD. [3]
- (b) Calculate the **total** volume of the barn. [3]

(This question continues on the following page)

(Question 1 continued)

The roof was built using metal supports. Each support is made from **five** lengths of metal AE, ED, AD, EM and MN, and the design is shown in the following diagram.

diagram not to scale



$ED = 7\text{ m}$, $AD = 10\text{ m}$ and angle $ADE = 15^\circ$.

M is the midpoint of AD.

N is the point on ED such that MN is at right angles to ED.

(c) Calculate the length of MN. [2]

(d) Calculate the length of AE. [3]

Farmer Brown believes that N is the midpoint of ED.

(e) Show that Farmer Brown is incorrect. [3]

(f) Calculate the **total** length of metal required for one support. [4]

2. [Maximum mark: 16]

On one day 180 flights arrived at a particular airport. The distance travelled and the arrival status for each incoming flight was recorded. The flight was then classified as on time, slightly delayed, or heavily delayed.

The results are shown in the following table.

		Distance travelled			TOTAL
		At most 500 km	Between 500 km and 5000 km	At least 5000 km	
Arrival Status	On time	19	17	16	52
	Slightly delayed	13	18	14	45
	Heavily delayed	28	15	40	83
	TOTAL	60	50	70	180

A χ^2 test is carried out at the 10% significance level to determine whether the arrival status of incoming flights is independent of the distance travelled.

- (a) State the alternative hypothesis. [1]
- (b) Calculate the expected frequency of flights travelling at most 500 km and arriving slightly delayed. [2]
- (c) Write down the number of degrees of freedom. [1]
- (d) Write down
 - (i) the χ^2 statistic;
 - (ii) the associated p -value. [3]

(This question continues on the following page)

(Question 2 continued)

The critical value for this test is 7.779.

- (e) State, with a reason, whether you would reject the null hypothesis. [2]

A flight is chosen at random from the 180 recorded flights.

- (f) Write down the probability that this flight arrived on time. [2]

- (g) Given that this flight was not heavily delayed, find the probability that it travelled between 500 km and 5000 km. [2]

Two flights are chosen at random from those which were slightly delayed.

- (h) Find the probability that each of these flights travelled at least 5000 km. [3]

3. [Maximum mark: 11]

Give your answers to parts (b), (c) and (d) to the nearest whole number.

Harinder has 14 000 US Dollars (USD) to invest for a period of five years. He has two options of how to invest the money.

Option A: Invest the full amount, in USD, in a fixed deposit account in an American bank.

The account pays a nominal annual interest rate of $r\%$, **compounded yearly**, for the five years. The bank manager says that this will give Harinder a return of 17 500 USD.

(a) Calculate the value of r . [3]

Option B: Invest the full amount, in Indian Rupees (INR), in a fixed deposit account in an Indian bank. The money must be converted from USD to INR before it is invested.

The exchange rate is 1 USD = 66.91 INR.

(b) Calculate 14 000 USD in INR. [2]

The account in the Indian bank pays a nominal annual interest rate of 5.2% **compounded monthly**.

(c) Calculate the amount of this investment, in INR, in this account after five years. [3]

Harinder chose option B. At the end of five years, Harinder converted this investment back to USD. The exchange rate, at that time, was 1 USD = 67.16 INR.

(d) Calculate how much **more** money, in USD, Harinder earned by choosing option B instead of option A. [3]

4. [Maximum mark: 14]

Consider the function $f(x) = \frac{48}{x} + kx^2 - 58$, where $x > 0$ and k is a constant.

The graph of the function passes through the point with coordinates (4, 2).

(a) Find the value of k . [2]

(b) Using your value of k , find $f'(x)$. [3]

P is the minimum point of the graph of $f(x)$.

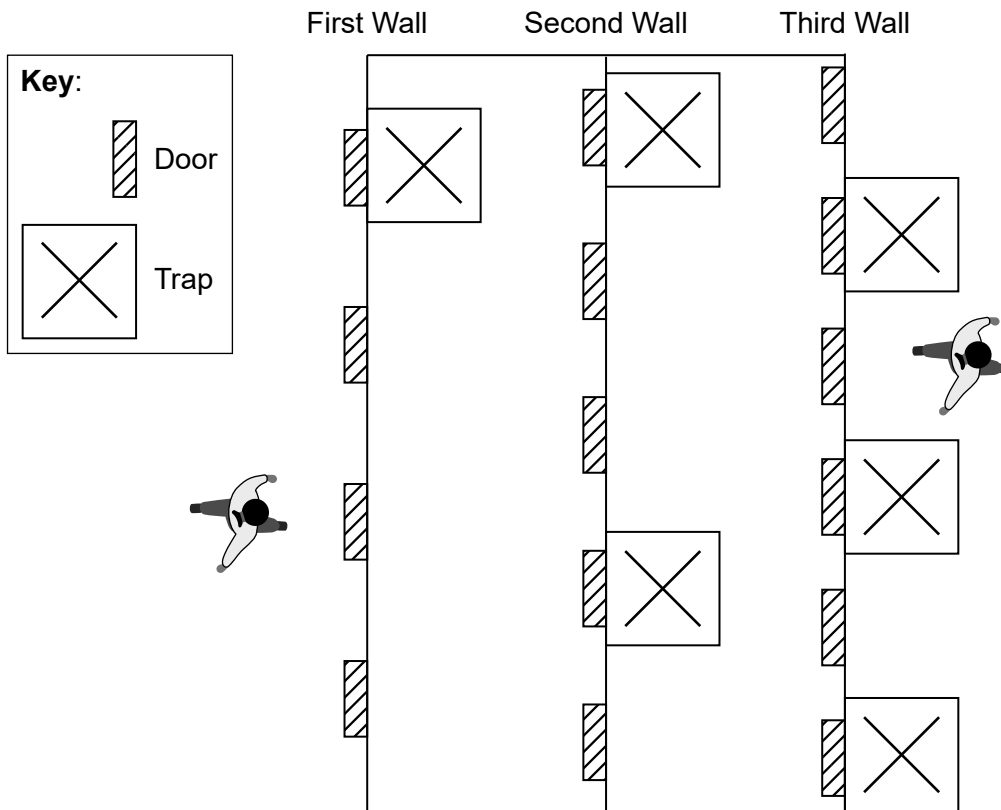
(c) **Use your answer** to part (b) to show that the minimum value of $f(x)$ is -22 . [3]

(d) Write down the **two** values of x which satisfy $f(x) = 0$. [2]

(e) Sketch the graph of $y = f(x)$ for $0 < x \leq 6$ and $-30 \leq y \leq 60$.
Clearly indicate the minimum point P and the x -intercepts on your graph. [4]

5. [Maximum mark: 15]

Contestants in a TV gameshow try to get through three walls by passing through doors without falling into a trap. Contestants choose doors at random. If they avoid a trap they progress to the next wall. If a contestant falls into a trap they exit the game before the next contestant plays. Contestants are not allowed to watch each other attempt the game.



The first wall has four doors with a trap behind one door.

Ayako is a contestant.

- (a) Write down the probability that Ayako avoids the trap in this wall. [1]

Natsuko is the second contestant.

- (b) Find the probability that only one of Ayako and Natsuko falls into a trap while attempting to pass through a door **in the first wall**. [3]

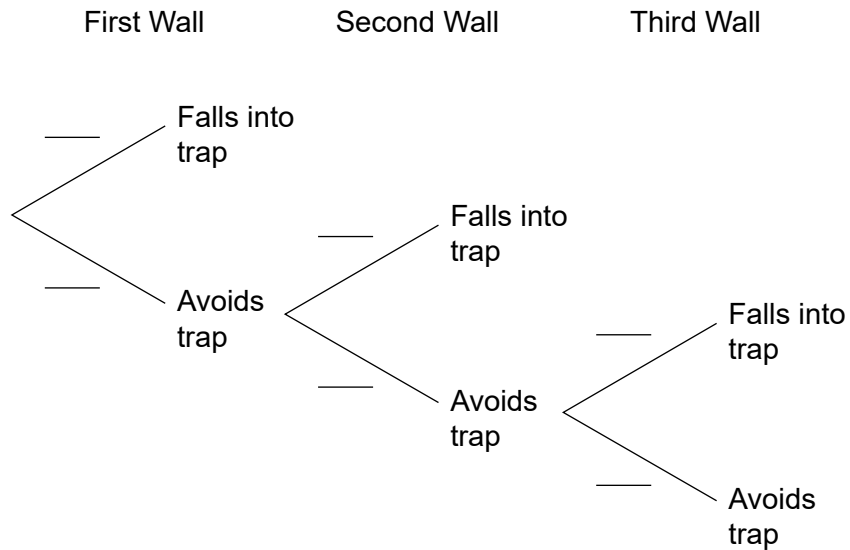
The second wall has five doors with a trap behind two of the doors.

The third wall has six doors with a trap behind three of the doors.

(This question continues on the following page)

(Question 5 continued)

The following diagram shows the branches of a probability tree diagram for a contestant in the game.



(c) **Copy** the probability tree diagram and write down the relevant probabilities along the branches. [3]

(d) A contestant is chosen at random. Find the probability that this contestant

(i) fell into a trap while attempting to pass through a door in the second wall;

(ii) fell into a trap. [5]

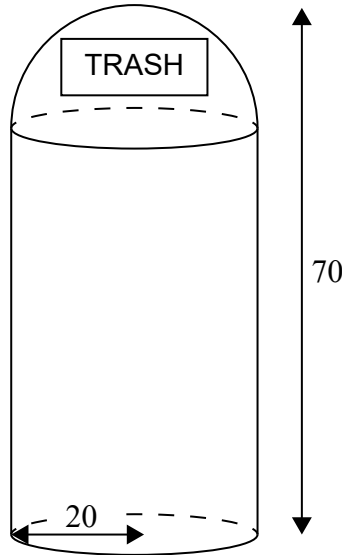
120 contestants attempted this game.

(e) Find the expected number of contestants who fell into a trap while attempting to pass through a door in the third wall. [3]

6. [Maximum mark: 16]

A manufacturer makes trash cans in the form of a cylinder with a hemispherical top. The trash can has a height of 70 cm. The base radius of both the cylinder and the hemispherical top is 20 cm.

diagram not to scale

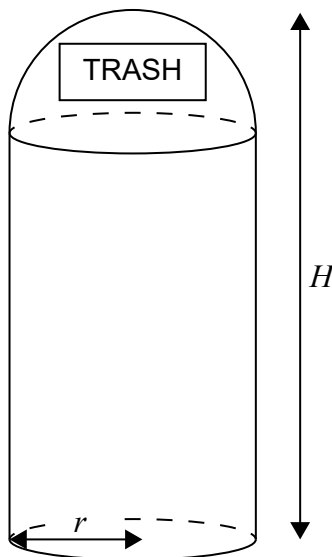


(a) Write down the height of the cylinder. [1]

(b) Find the total volume of the trash can. [4]

A designer is asked to produce a new trash can. The new trash can will also be in the form of a cylinder with a hemispherical top. This trash can will have a height of H cm and a base radius of r cm.

diagram not to scale



(This question continues on the following page)

(Question 6 continued)

There is a design constraint such that $H + 2r = 110$ cm.

The designer has to maximize the volume of the trash can.

(c) Find the height of the **cylinder**, h , of the new trash can, in terms of r . [2]

(d) Show that the volume, V cm³, of the new trash can is given by

$$V = 110\pi r^2 - \frac{7}{3}\pi r^3. \quad [3]$$

(e) Using your graphic display calculator, find the value of r which maximizes the value of V . [2]

The designer claims that the new trash can has a capacity that is at least 40% greater than the capacity of the original trash can.

(f) State whether the designer's claim is correct. Justify your answer. [4]
